I SIMPÓSIO DE MODELAGEM COMPUTACIONAL EM CIÊNCIA E TECNOLOGIA

FREE VIBRATION ANALYSIS OF A ROTATING TAPERED BEAM

Juan Gabriel Queiroz Blanche (Curso de Engenharia Mecânica – UFPI);
Simone dos Santos Hoefel (Orientadora, Curso de Engenharia Mecânica – UFPI);
Laboratório de Métodos de Modelagem Computacional – LaMeC;
juanblanche2@gmail.com.

Introduction

Rotating beams are widely found in several engineering structures such as wind turbine blades, industrial fans and gas turbine engine. Blade is one of the most critical mechanical component in rotating machines, as result, the behavior of rotating beams structures has received a lot of attention. Therefore, different configurations of blades was investigated with classical theory of beam by many researchers, such as Euler-Bernoulli and Timoshenko. Since blades rarely has uniform area along the length, a non-prismatic analysis becomes more accurate and reliable motivating several authors to investigate the effect of tapering on rotating beams.

In this paper, the vibration frequency of a tapered rotating beam is investigated. The finite element method is used, a non-constant flexural rigidity and mass per unit over the element is developed using a cubic polynomial. The effects of taper ratio and rotational speed are analyzed and the mode shapes are determined for some values of taper parameters. The results obtained shown that Rayleigh-Ritz method for Finite Elements finds the natural frequencies of a rotating tapered cantilever Euler-Bernoulli beam with high precision.

Methodology

Tapered Euler-Bernoulli model

The governing differential equations of motion of the rotating tapered beam in free vibration are derived for the general case by applying Hamilton's principle which requires the expressions for potential and kinetic energies of the beam as fundamental prerequisites. Considering an element beam with constant rotation speed in a plane normal to the axis, the potential energy and kinetic energy are given, respectively, by GANGULI (2017):

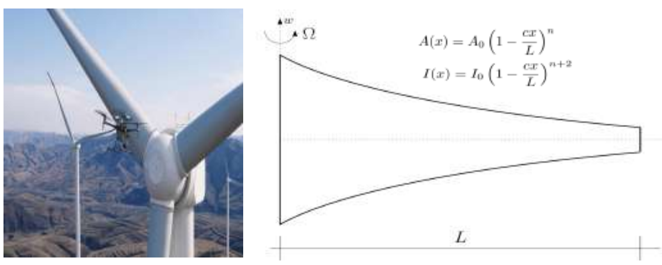

Figure 1: Model of a rotating tapered beam.

$$U = \int_0^L \left[EI(x)\left(\frac{\partial^2 w}{\partial x^2}\right)^2\right]dx + \frac{1}{2}\int_0^L \left[P(x)\left(\frac{\partial w}{\partial x}\right)^2\right]dx \tag{1}$$

$$T = \frac{1}{2}\int_0^L \left[\rho A(x)\left(\frac{\partial w}{\partial t}\right)^2\right]dx\,. \tag{2}$$

Where E is the modulus of elasticity, ρ, is the mass per unit volume, $I(x)$, the moment of inertia of cross section varying along the beam length, $A(x)$, the cross-sectional area varying along the beam length and A_0 and I_0 are, respectively, the initial area and moment of inertia.

For a constant rotate speed Ω, the centrifugal force will given by:

$$P(x) = \int_x^L \rho A(x)\Omega^2 x\, dx. \tag{3}$$

Finite element formulation

Nonuniform beam element consists of two nodes and each node has two degrees of freedom: w, the total deflection, θ, the slope due to bending, r is the element axis and $\xi = \frac{r}{l}$.

Figure 2 shows the variation of the cross-sectional area, moment of inertia and centrifugal force in a generic element i. Where N is the total number of elements and m is the element before the i-th element analyzed.

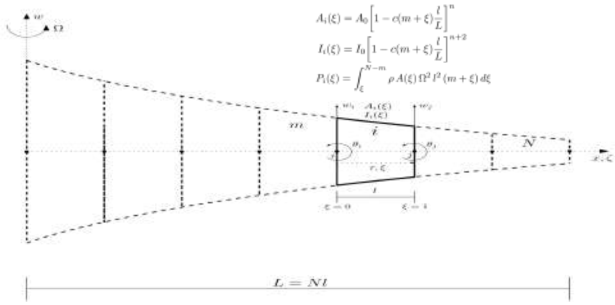

Figure 2: Finite element model for a rotating tapered beam.

Results and discussion

This section presents numerical examples for rotating tapered beam subjected to boundary conditions clamped-free. In order to investigate the influence the effects of rotational speed parameter (η) and taper ratio (c), natural frequencies are calculated to various values of (η) and (c). Table 1 shows the results obtained by present work using FEM and the results obtained by RASAJEKARAN (2013) and BANERJEE et al. (2006), using Differential Transformation Method(DTM) and Dynamic Stiffness Method (DSM).

([1]) RASAJEKARAN (2013)
([2]) BANERJEE (2006)

$\eta = \Omega L^2\sqrt{\frac{\rho A_0}{EI_0}}$	$\omega_n(rad/s)$ FEM 70e.			DTM[1]			DSM[2]		
	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
0	3.8238	18.3173	47.2648	3.8238	18.3173	47.2648	3.82379	18.3173	47.2648
4	5.8788	20.6852	49.6456	5.8788	20.6852	49.6456	5.87877	20.6851	49.6456
8	9.55396	26.5437	56.1595	9.554	26.5437	56.1595	9.55396	26.5437	56.1595
10	11.5015	30.1827	60.5639	11.5015	30.1829	60.5643	11.5015	30.1827	60.5639

Table 1: Influence of angular velocity on the natural frequencies of vibration of a tapered beam with $c = 0.5$ and $n = 1$.

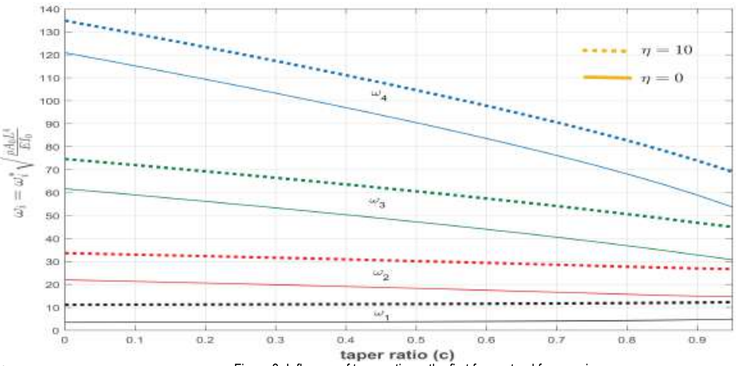

Figure 3: Influence of taper ratio on the first four natural frequencies.

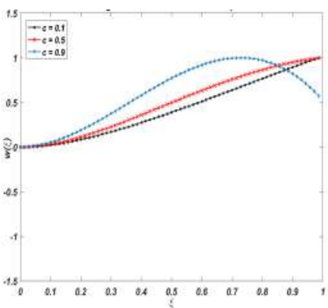

Figure 4 –First mode shape of a rotating tapered beam (with $\eta = 10$ and $n = 1$).

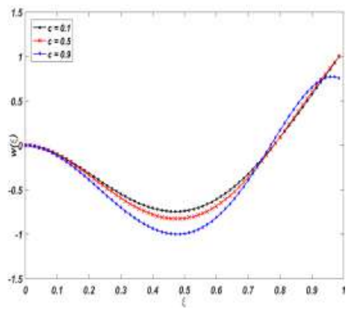

Figure 5 –Second mode shape of a rotating tapered beam (with $\eta = 10$ and $n = 1$).

Conclusion

We have investigated the effects of taper ration and rotational speed on classical beam theory. Results shows that natural frequencies increases as angular velocity becomes higher. This occurs due to increased stiffness of the system caused by the centrifugal force.

Futhermore, it was observed an interesting behavior related with taper ratio parameter. First frequency increases while frequencies for higher modes decreases as taper ratio arises. Finally, it was shown on numerical examples that finite elements results were in well agreement with literature.

References

BANERJEE, J.R.; SU, H.; JACKSON, D.R. "Free vibration of rotating beams using the dynamic stiffness method. **Journal of Sound and Vibrations**, Vol. 298, p.1034-1054, 2006.

GANGULI, R. **Finite Element Analysis of Rotating Beams**. Springer, 2017.

RAJASEKARAN, S. Differential transformation and differential quadrature methods for centrifugally stiffened axially functionally graded tapered beams. **International Journal of Mechanical Sciences**, Vol. 74, p. 15-31, 2013.

LAMEC
LABORATÓRIO DE MÉTODOS DE MODELAGEM COMPUTACIONAL
UNIVERSIDADE FEDERAL DO PIAUÍ