I SIMPÓSIO INTERNACIONAL DE MODELAGEM COMPUTACIONAL EM CIÊNCIA E TECNOLOGIA

VIBRATION RESPONSE OF A BEAM-COLUMN ON PASTERNAK FOUNDATION.

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Introduction

Several researchers over the years have sought describe and analyze problems of soil-structure interaction. For example, Arboleda-Monsalve et al. (2008) used the finite element method (FEM) to analyze the dynamic of a Timoshenko beam-column resting on elastic foundation with general constraints ends, and Abohadima et al. (2015) investigated the free and forced vibration of an axial-loaded Timoshenko beam resting on a elastic foundation using the Recursive Differentiation Method (RDM). Further, Soares et al. (2017), also using the FEM, investigated the dynamic behave of a Timoshenko beam resting on a Pasternak foundation.

In this context, the purpose of this work is investigate the dynamic behavior of an axial-loaded Timoshenko beam resting on a two-parameter elastic foundation. FEM is developed and compared with the analytical solution. Foundation parameters and the axial load contributions are investigated in several numerical examples for beams with different end restraints.

Methodology

Axial-loaded Beam On Elastic Foundation Formulation

Differential Equations (1) and (2) express the motion for a free vibration beam, accordingly to the Timoshenko formulation taking into account the foundation and the axial load, as show in Fig. 1, where L is the length of the beam, P the axial load, kw the elastic linear spring parameter and G_p the shear layer parameter (Wang and Stephens, 1977; Wu, 2013).

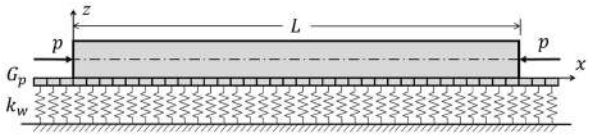

Figure 1: Sketch of an axial-loaded beam resting on a Pasternak foundation (image elaborated by the authors).

$$EI\left(1+\frac{G_p-p}{\kappa GA}\right)\frac{\partial^4 w(x,t)}{\partial x^4}+\left(\rho A+\frac{\rho I k_w}{\kappa GA}\right)\frac{\partial^2 w(x,t)}{\partial t^2}-\left(\frac{EIk_w}{\kappa GA}+G_p-p\right)\frac{\partial^2 w(x,t)}{\partial x^2}-\rho I\left(1+\frac{E}{\kappa G}+\frac{G_p-p}{\kappa GA}\right)\frac{\partial^4 w(x,t)}{\partial x^2\partial t^2}+\frac{\rho^2 I}{\kappa G}\frac{\partial^4 w(x,t)}{\partial t^4}+k_w w(x,t)=0, \tag{1}$$

$$EI\left(1+\frac{G_p-p}{\kappa GA}\right)\frac{\partial^4 \psi(x,t)}{\partial x^4}+\left(\rho A+\frac{\rho I k_w}{\kappa GA}\right)\frac{\partial^2 \psi(x,t)}{\partial t^2}-\left(\frac{EIk_w}{\kappa GA}+G_p-p\right)\frac{\partial^2 \psi(x,t)}{\partial x^2}-\rho I\left(1+\frac{E}{\kappa G}+\frac{G_p-p}{\kappa GA}\right)\frac{\partial^4 \psi(x,t)}{\partial x^2\partial t^2}+\frac{\rho^2 I}{\kappa G}\frac{\partial^4 \psi(x,t)}{\partial t^4}+k_w \psi(x,t)=0, \tag{2}$$

where, E is the Young's modulus, G the shear modulus, I the moment of inertia, ρ the mass density, A the cross-sectional area, t the time, κ the shear factor, $w(x,t)$ is vertical displacement and $\psi(x,t)$ is total slope.

Finite Element Method

Consider a uniform axial-loaded Timoshenko beam element on Pasternak Foundation with length $2a$ as showed in Fig. 2. The beam element consists of two nodes and each node has two degrees of freedom: w, the total displacement and ψ, the total slope.

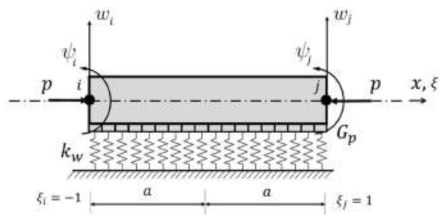

Figure 2: Beam element (image elaborated by the authors).

Applying the Rayleigh-Ritz method using polynomial functions approximations and applying the boundary conditions at the points $\xi_i = -1$ and $\xi_j = 1$, the shape functions and the vector of nodal coordinates can be expressed as:

$$[N(\xi)]_w^T = \frac{1}{4(1+3\beta)}\begin{bmatrix} 2-3(1+2\beta)\xi+6\beta+\xi^3 \\ a(1-\xi-(1+3\beta)\xi^2+3\beta+\xi^3) \\ 2+3(1+2\beta)\xi+6\beta-\xi^3 \\ a(-1-\xi+(1+3\beta)\xi^2-3\beta+\xi^3) \end{bmatrix}, \tag{7}$$

$$[N(\xi)]_\psi^T = \frac{1}{4(1+3\beta)}\begin{bmatrix} (-3+3\xi^2)/a \\ -1-(2+6\beta)\xi+6\beta+3\xi^3 \\ (3-3\xi^2)/a \\ -1+(2+6\beta)\xi+6\beta+3\xi^3 \end{bmatrix} \text{ and } \{v\}_e^T = [w_i \quad \psi_i \quad w_j \quad \psi_j], \tag{8; 9}$$

where $\beta = EI/(\kappa GAa^2)$.

Therefore, the elementary stiffness and mass matrices, $[k_e]$ and $[m_e]$, respectively are given by:

$$[k_e] = \frac{EI}{a}\int_{-1}^{1}[N'(\xi)]_\psi^T[N'(\xi)]_\psi d\xi + \frac{\kappa GA}{a}\int_{-1}^{1}\left[[N'(\xi)]_w - a[N(\xi)]_\psi\right]^T\left[[N'(\xi)]_w - a[N(\xi)]_\psi\right]d\xi + k_w a\int_{-1}^{1}[N(\xi)]_w^T[N(\xi)]_w d\xi + \frac{G_p}{a}\int_{-1}^{1}[N'(\xi)]_w^T[N'(\xi)]_w d\xi + \frac{P}{a}\int_{-1}^{1}[N'(\xi)]_w^T[N'(\xi)]_w d\xi \tag{11}$$

and

$$[m_e] = \rho Aa\int_{-1}^{1}[N(\xi)]_w^T[N(\xi)]_w d\xi + \rho Ia\int_{-1}^{1}[N(\xi)]_\psi^T[N(\xi)]_\psi d\xi\ . \tag{12}$$

Numerical Results And Discussion

In this section, a numerical example presented by Abohadima et al. (2015) is analyzed. A uniform beam of uniform cross-section area such that $\nu = 1/4$, $\mathrm{E/G} = 2.5$, the shear factor given by $\kappa = 2/3$, $\rho = 2500kg/m^3$, $I = 0{,}05^2 m^4$ and $A = 1m^2$. First three frequencies parameters obtained for immersed and non-immersed cases are presented on Tab. 1. Finite elements (discretization with 30 and 50 elements) results were in well agreement with literature (Abohadima et al., 2015).

Table 1: First three frequencies parameters for a hinged-hinged beam (table elaborated by the authors).

Parameters				30 ele.	50 ele.	Exact[1]
n	e	p				
0	0	0	b_1	2.866	2.866	2.866
			b_2	4.925	4.923	4.922
			b_3	6.454	6.449	6.446
0.6	0	0	b_1	1.862	1.862	1.863
			b_2	4.387	4.384	4.384
			b_3	5.932	5.926	5.923
0.6	$0.6\pi^4$	0	b_1	2.866	2.866	2.866
			b_2	4.540	4.538	4.538
			b_3	5.997	5.991	5.988
0.6	$0.6\pi^4$	π^2	b_1	3.555	3.555	3.555
			b_2	5.296	5.294	5.294
			b_3	6.785	6.779	6.777

[1](Abohadima et al., 2015).

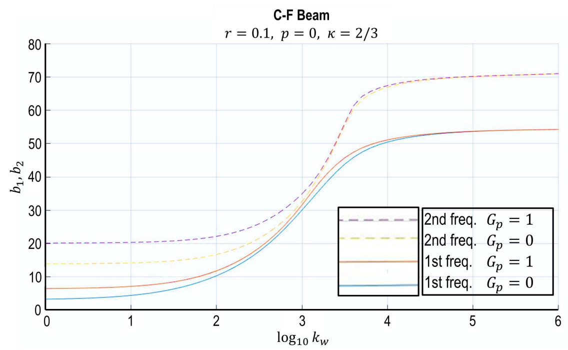

Figure 3: First two frequencies parameters for a Clamped-Free beam on a Pasternak foundation.

Conclusions

Notice that Tab. 1 show that the presence of axial load decreases the frequency parameters b_i. This behavior occurs because the transverse component of the axial load causes an increase in shear force. Also, is observed the influence of foundation into free vibration of a hinged-hinged beam subjected to axial load. The addition of the first foundation parameter causes a significant increase in the frequency parameter in the first mode, although become less significant for higher modes. Further, results for second foundation parameter (Pasternak foundation) presents a remarkable increase for all modes of vibration showed, being superior than those presented by the first foundation parameter. In addition, the variations in the frequency parameter decreases as the mode number increases. For higher modes, as expected, the difference between FEM and analytical solutions decreases as the number of elements is increased. Finally, the major error was 0,152%, perceived into third frequency using 30 elements.

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