FREE VIBRATION OF A BEAM-COLUMN ON TWO PARAMETER ELASTIC FOUNDATION

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Introduction

The behavior of the Euler-Bernoulli beam on elastic foundation has been extensively studied in the modern engineering. There are many applications for beam on elastic foundation mainly in mechanical and civil engineering, like shafts supported on ball, roller, or journal bearings, vibrating machines on elastic foundations, network of beams in the construction of floor systems for ships, buildings, submerged floating tunnels, buried pipelines, railroad tracks etc.

The purpose of this work is to analyze the influence of the axial load on the dynamic response of an axial-loaded Euler-Bernoulli beam resting on two-parameter foundation. Some examples presents the influence of the axial load and the foundation in the natural frequencies of the beam. Also, mode shapes were determined for some classical boundary conditions. The results obtained by FEM are discussed and compared with the exact solution.

Methodology

Classical Theory

Consider an Euler-Bernoulli beam of length L, resting on two-parameter foundation subject to an axial load p as shows the scheme in the Fig. 1. The parameters of the foundation are given by k_1 and k_2, that are the elastic linear springs coefficient and the shear layer stiffness coefficient, respectively.

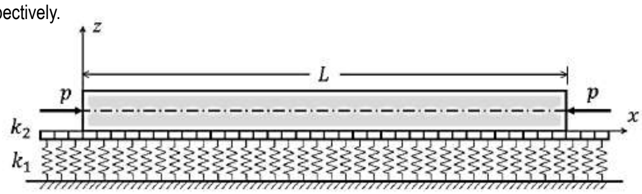

Figure 1: Euler-Bernoulli beam resting on two-parameter foundation subject to an axial load.

The potential and kinetic energy of the system can be expressed as (Soares and Hoefel, 2015):

$$U = \frac{1}{2}\int_0^L EI\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx + \frac{1}{2}\int_0^L k_1 w^2 dx + \frac{1}{2}\int_0^L k_2\left(\frac{\partial w}{\partial x}\right)^2 dx + \frac{1}{2}\int_0^L p\left(\frac{\partial w}{\partial x}\right)^2 dx, \quad (1)$$

$$T = \frac{1}{2}\int_0^L \rho A\left(\frac{\partial w}{\partial t}\right)^2 dx, \quad (2)$$

where E is the Young's modulus, I the moment of inertia of the cross-section about the y axis, ρ the mass density, A the cross-section area and w the displacement dependent of the axial location x and of the time t.

Using the Hamilton's principle and after some manipulations, is obtained the differential equation for free vibration response:

$$EI\left(\frac{\partial^4 w}{\partial x^4}\right) + \rho A\left(\frac{\partial^2 w}{\partial t^2}\right) + k_1 w - k_2\left(\frac{\partial^2 w}{\partial x^2}\right) + p\left(\frac{\partial^2 w}{\partial x^2}\right) = 0. \quad (3)$$

Finite Element Formulation

Consider an uniform axial-loaded Euler-Bernoulli beam element on two-parameter foundation as shown in Fig. 2. The beam element consists of two nodes and each node has two degrees of freedom: w, the total deflection, and θ, the slope due to bending. Applying the Rayleigh-Ritz method, the displacement w is approximated for a polynomial function with four constants, so that a_i are constants (Petyt, 2010).

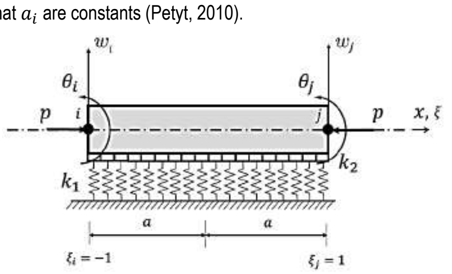

Figure 2: Beam element.

Using the non-dimensional coordinate ξ, the element length $2a$, and applying the boundary conditions on the points $\xi = -1$ and $\xi = 1$, the matrix form of the displacement can be written as: $w = [\mathbf{N}(\xi)]\{\mathbf{v}\}_e$, where $[\mathbf{N}(\xi)]$ are the shape functions and $\mathbf{v}$ is the vector of nodal coordinates. The subscript e represents expressions for a single element. Therefore, the shape functions and the vector of nodal coordinates, can be expressed as:

$$[\mathbf{N}(\xi)]^T = \frac{1}{4}\begin{bmatrix} 2-3\xi+\xi^3 \\ a(1-\xi-\xi^2+\xi^3) \\ 2+3\xi-\xi^3 \\ a(-1-\xi+\xi^2+\xi^3) \end{bmatrix}, \{\mathbf{v}\}_e^T = [w_1 \quad \theta_1 \quad w_2 \quad \theta_2]. \quad (4)$$

The potential and kinetic energy for an element length $2a$ are given by (Azevedo *et al.*, 2016):

$$\mathbf{U}_e = \frac{1}{2}\frac{EI}{a}\int_{-1}^1 \left(\frac{\partial^2 w}{\partial \xi^2}\right)^2 d\xi + \frac{1}{2}k_1 a\int_{-1}^1 w^2 d\xi + \frac{1}{2}\frac{k_2}{a}\int_{-1}^1 \left(\frac{\partial w}{\partial \xi}\right)^2 d\xi + \frac{1}{2}\frac{p}{a}\int_{-1}^1 \left(\frac{\partial w}{\partial \xi}\right)^2 d\xi, \quad (5)$$

$$\mathbf{T}_e = \frac{1}{2}\rho A a\int_{-1}^1 \left(\frac{\partial w}{\partial t}\right)^2 d\xi. \quad (6)$$

Substituting the Eq. (4) into Eqs. (5) and (6). The elementary stiffness and elementary inertia matrix are given by:

$$\begin{aligned}[\mathbf{k}]_e = {} & \frac{EI}{a}\int_{-1}^1 [\mathbf{N}(\xi)'']^T[\mathbf{N}(\xi)'']d\xi + k_1 a\int_{-1}^1 [\mathbf{N}(\xi)]^T[\mathbf{N}(\xi)]d\xi + \\ & \frac{k_2}{a}\int_{-1}^1 [\mathbf{N}(\xi)']^T[\mathbf{N}(\xi)']d\xi + \frac{p}{a}\int_{-1}^1 [\mathbf{N}(\xi)']^T[\mathbf{N}(\xi)']d\xi,\end{aligned} \quad (7)$$

$$[\mathbf{m}]_e = \rho A a\int_{-1}^1 [\mathbf{N}(\xi)]^T[\mathbf{N}(\xi)]d\xi. \quad (8)$$

Numerical Results and Discussion

This section present a numerical example for various load (P), elastic (K_1) and shear layer stiffness (K_2) coefficients. Consider an uniform Euler-Bernoulli beam with hinged-hinged ends, such that $E/G = 2.5$, $\rho = 2500kg/m^3$, $r_g = 0.05m$, $I = 0.05^2m^4$, $A = 1m^2$, $\nu = 0.25$ and $L = 0.5m$. Frequency parameter calculated using FEM (discretizations with 10 and 20 elements) are compared with the exact solution presented by Yokoyama (1996) for the first three vibration modes as shown in the Tab. 1.

Table 1: Values of frequency parameters of the first three vibration modes .

Parameters			b_1			b_2			b_3		
P	K_1	K_2	10 ele.	20 ele.	Exact[(1)]	10 ele.	20 ele.	Exact[(1)]	10 ele.	20 ele.	Exact[(1)]
0	0	0	9.87	9.87	9.87	39.48	39.48	39.48	88.87	88.83	88.83
0.6	0	0	6.24	6.24	6.24	36.4	36.4	36.4	85.86	85.82	85.81
0.6	$0.6\pi^4$	0	9.87	9.87	9.87	37.2	37.19	37.19	86.2	86.15	86.15
0.6	$0.6\pi^4$	π^2	13.96	13.96	13.96[(2)]	42.11	42.11	42.25[(2)]	91.15	91.1	92.69[(2)]

(1) (Yokoyama, 1996)
(2) (Results of Yokoyama using FEM.)

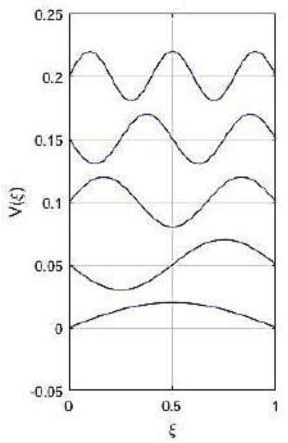

Figure 3 – Lowest five mode shapes with hinged-hinged ends.

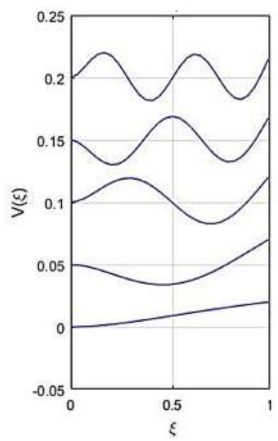

Figure 4 – Lowest five mode shapes with Clamped-Free ends.

Conclusion

The presence of an axial compressive load decrease the frequency parameter, this occurs because the contribution of the vertical component of the axial load cause an increase in the shear stiffness. The addition of the Winkler foundation, cause a significant increase in the frequency parameter in the first mode, although become less significant for higher modes. Finally, for the two-parameter foundation, the frequency parameters presents a remarkable increase for all modes of vibration showed, being superior than those presented by the Winkler foundation parameter.

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