NONLOCAL FINITE ELEMENT ANALISYS OF EULER-BERNOULLI NANOBEAMS

Aldemar Pontes de Siqueira Neto (Curso de Engenharia Mecânica − UFPI);
Simone dos Santos Hoefel (Orientadora, Curso de Engenharia Mecânica − UFPI);
Laboratório de Métodos de Modelagem Computacional − LaMeC;
aldemarpsn@gmail.com

Introduction

Nanostructured materials present outstanding features like elevated modulus of elasticity and yield strength, high flexibility and conductivity. This materials are widely applied to nanoelectronics, nanodevices, nanosensors and nanocomposites.

Various nonclassical elasticity theories were developed to study the mechanical behaviour of nanostructures, among them, nonlocal elasticity theory (ERINGEN, 1983) is the most used. In this theory, a nonlocal parameter which accounts for small scale effects is included in the constitutive equations. Carbon nanotubes are an extensively studied nanostructure often modelled as an Euler-Bernoulli nanobeam.

In the present work, governing equations for bending, buckling and free vibration of an Euler-Bernoulli nanobeam are presented. A finite element model is developed to investigate the influence of nonlocal parameter, slenderness, rotational inertia and boundary conditions on deflections, buckling loads and natural frequencies. Numerical results obtained are discussed and compared with results obtained by other researchers.

Methodology

Governing equations of an Euler-Bernoulli beam

The governing equations for an local Euler-Bernoulli beam are given by Reddy (2007):

$$\frac{\partial N}{\partial x}+f=I_0\frac{\partial^2 u}{\partial x^2} \quad \text{and} \tag{1}$$

$$\frac{\partial^2 M^E}{\partial x^2}+q\frac{\partial}{\partial x}\left(\bar{N}^E\frac{\partial w^E}{\partial x}\right)=I_0\frac{\partial^2 w^E}{\partial x^2}-I_2\frac{\partial^4 w^E}{\partial x^2\partial t^2}, \tag{2}$$

where u is the axial displacement, w is the transverse displacement, f and q are the longitudinal and transverse distributed loads, $\bar{N}^E$ is the applied axial compressive load, the stress resultants N e M^E are defined as

$$N=\int_A \sigma_{xx}\,dA\,,\quad M^E=\int_A z\sigma_{xx}\,dA, \tag{3}$$

where σ_{xx} is the normal stress and the mass inertias are given by:

$$I_0=\int_A \rho\,dA\,,\quad I_2=\int_A \rho z^2\,dA, \tag{4}$$

where ρ is the specific mass of the beam.

Nonlocal elasticity theory and governing equations for a nonlocal Euler-Bernoulli

The differential formulation of the nonlocal elasticity theory is given by Eringen (1983):

$$\left(1-e_0^2 l_i^2\nabla^2\right)\sigma=t, \tag{5}$$

where e_0 is the nonlocal parameter, l_i is the internal characteristic length, σ is the nonlocal stress tensor and t is the macroscopic stress tensor at a given point. For an Euler-Bernoulli beam this relation is reduced to the following constitutive equation:

$$\sigma_{xx}-\mu\frac{\partial^2\sigma_{xx}}{\partial x^2}=E\varepsilon_{xx}, \tag{6}$$

where $\mu=e_0^2 l_i^2$, ε_{xx} is the normal strain and E is the material Young modulus. This relation can be used to write the following governing equations in the axial and transverse directions for a nonlocal Euler-Bernoulli beam:

$$\frac{\partial}{\partial x}\left(EA\frac{\partial u}{\partial x}\right)+f-\mu\frac{\partial^2 f}{\partial x^2}=I_0\left(\frac{\partial^2 u}{\partial t^2}-\mu\frac{\partial^4 u}{\partial x^2\partial t^2}\right), \tag{7}$$

$$\frac{\partial^2}{\partial x^2}\left(-EI\frac{\partial^2 w^E}{\partial x^2}\right)+\mu\frac{\partial^2}{\partial x^2}\left[\frac{\partial}{\partial x}\left(\bar{N}^E\frac{\partial w^E}{\partial x}\right)-q+I_0\frac{\partial^2 w^E}{\partial x^2}-I_2\frac{\partial^4 w^E}{\partial x^2\partial t^2}\right]$$
$$+q-\frac{\partial}{\partial x}\left(\bar{N}^E\frac{\partial w^E}{\partial x}\right)=I_0\frac{\partial^2 w^E}{\partial t^2}-I_2\frac{\partial^4 w^E}{\partial x^2\partial t^2}. \tag{8}$$

Finite element formulation

The finite element model is developed based upon the Hamilton's principle from the governing equations and the nonlocal constitutive relation. A beam element with two nodes and three degrees of freedom per node is used in the domain discretization.

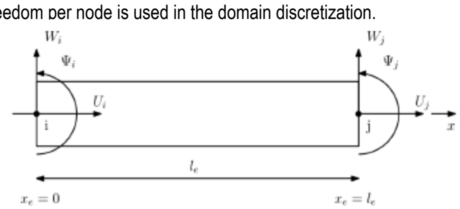

Figure 1 – Nanobeam element.

Numerical Results and Discussion

Table 1: Beam properties.

Young modulus (E)	Specific mass (ρ)	Length (L)	Poisson's coefficient (ν)
$30\,GPa$	$1\,kg/m^3$	$10\,m$	0,3

Table 2: Comparison of non-dimensional quantities for a simply supported nanobeam.

L/h	μ	$\bar{w}^{(1)}$	$\bar{w}$ (present)	$\bar{N}_{cr}^{(1)}$	$\bar{N}_{cr}$ (present)	$\bar{\omega}_1^{(1)}$	$\bar{\omega}_1$ (present)
100	0	1,3021	1,3021	9,8696	9,8696	9,8696	9,8696
	1	1,4271	1,4271	8,9830	8,9830	9,4159	9,4159
	1,5	1,4896	1,4896	8,5969	8,5969	9,2113	9,2113
	2	1,5521	1,5521	8,2426	8,2426	9,0195	9,0195
20	0	1,3021	1,3021	9,8696	9,8696	9,8696	9,8696
	1	1,4271	1,4271	8,9830	8,9830	9,4159	9,4159
	1,5	1,4896	1,4896	8,5969	8,5969	9,2113	9,2113
	2	1,5521	1,5521	8,2426	8,2426	9,0195	9,0195

(1) PRADHAN (2012)

Table 3: Influence of nonlocal parameter μ and rotational inertia I_2 for various boundary conditions for a nanobeam with L/h = 20.

μ	C. C.	$\bar{w}$	$\bar{N}_{cr}$	$\bar{\omega}_1$	$\bar{\omega}_1$ I_2 present
0	Simply Supported	1,3021	9,8696	9,8696	9,8595
	Cantilever	12,5000	2,4674	3,5160	3,5143
	Clamped-Supported	0,5416	20,1907	15,4182	15,3997
	Clamped-Clamped	0,2604	39,4784	22,3733	22,3447
1	Simply Supported	1,4271	8,9830	9,4159	9,4062
	Cantilever	12,0000	2,4080	3,5313	3,5295
	Clamped-Supported	0,5769	16,7989	14,5992	14,5803
	Clamped-Clamped	0,2604	28,3043	21,1090	21,0751

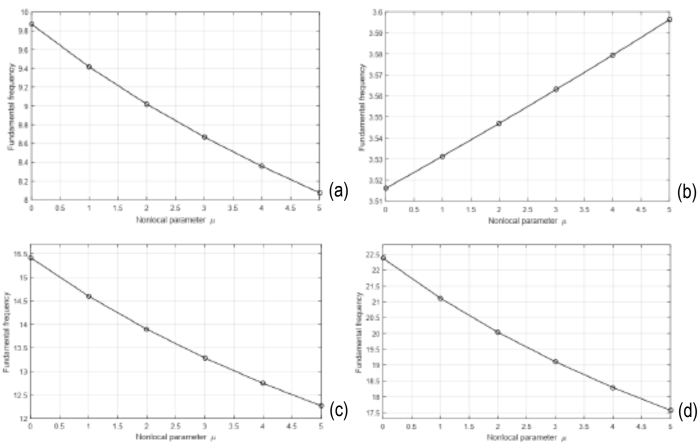

Figure 2 – Influence of nonlocal parameter μ in fundamental frequency for (a) simply supported, (b) cantilever, (c) clamped-supported and (d) clamped-clamped nanobeam with L/h = 100.

Conclusion

Numerical results obtained show that the inclusion of nonlocal effects in the Euler-Bernoulli theory increases the magnitudes of deflections and decreases buckling loads and natural frequencies. Slenderness ratio has no effect on the deflection results and the rotational inertia, when included, decreases fundamental frequencies. The effect of the parameters vary for different boundary conditions.

References

ANSARI, R; TORABI, J; NOROUZZADEH, A. Bending analysis of embedded nanoplates based on the integral formulation of Eringen's nonlocal theory using the finite element method. **Physica B: Condensed Matter**, Vol. 534, pp. 90−97, 2018.

DEMIR, Ç; CIVALEK, Ö. A new nonlocal FEM via Hermitian cubic functions for thermal vibration of nano beams surrounded by an elastic matrix. **Composite Structures,** Vol. 168, pp. 872−884, 2017.

ERINGEN, A Cemal. On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves. **Journal of applied physics**, Vol. 54, n. 9m

PRADHAN, S. Nonlocal finite element analysis and small scale effects of cnts with timoshenko beam theory. **Finite Elements in Analysis and Design**, Vol. 50, pp. 8−20, 2012.

REDDY, J. N. Nonlocal theories for bending, buckling and vibration of beams. **International Journal of Engineering Science**, Vol. 45, n. 2-8, pp. 4703−4710, 1983.

LAMEC LABORATÓRIO DE MÉTODOS DE MODELAGEM COMPUTACIONAL

UNIVERSIDADE FEDERAL DO PIAUÍ