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Periodic Structures for Vibration Attenuation in Tracks

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Introduction

Railway tracks are subject to dynamic loads that generate vibrations in rails, ballast, and underlying structures. These vibrations can cause noise, accelerate maintenance needs, and affect nearby infrastructure. Periodic structures offer a passive solution by creating band gaps - frequency ranges where wave propagation is suppressed. In rail systems, such effects can be achieved using components like pads or dampers with varying mass and stiffness.

This study analyzes how changes in the mass ratio between adjacent elements influence the formation and width of the first band gap, using analytical models and Wave Finite Element Method (WFEM) simulations.

Methodology

Railway tracks (Fig. 1) can be idealized as periodic systems when analyzing vibration propagation along structural elements such as rail pads, sleepers, or substructure layers. To model and understand these effects, the mathematical formulation of periodic structures is developed by integrating the Finite Element Method (FEM) with Floquet–Bloch theory, allowing for the analysis of wave propagation and attenuation in periodic media.

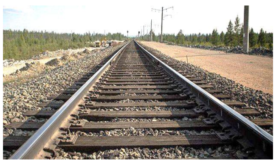

Fig. 1: Railway tracks
Fonte: AGICO Group*

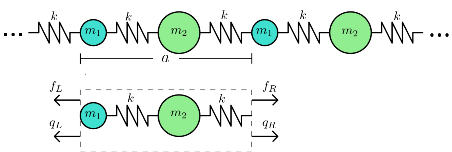

Fig 2: 1D Spring-mass periodic system and its unit cell interactions

To evaluate the specific influence of mass ratio ($\alpha = m_2 / m_1$) on the formation of band gaps, an infinite 1D spring–mass periodic system was adopted, composed of two alternating masses (m_1 and m_2) connected by springs of stiffness k, illustrated in Fig. 2. The equation of undamped harmonic motion is given by:

$$[\boldsymbol{K} - \omega^2 \boldsymbol{M}]\boldsymbol{q} = \boldsymbol{f},$$

where **K** and **M** denotes the stiffness and mass matrices, respectively, **q** the vector of degrees of freedom, **f** the vector of nodal forces, and $\boldsymbol{\omega}$ the angular frequency. Periodicity is imposed by the Floquet-Bloch theorem to reduce the entire infinite system to a single unit cell, relating the boundary parameters shown in Fig. 2 as follows:

$$\boldsymbol{q_R} = \boldsymbol{q_L}\lambda \quad \text{and} \quad \boldsymbol{f_R} = -\boldsymbol{f_L}\lambda\,.$$

The Bloch propagator is $\boldsymbol{\lambda} = e^{i\mu}$, is related to the propagation constant as $\mu = ka$, in which k represents the wavenumber.

Inverse Method imposes real values of propagation constants to obtain the angular frequencies. The following eigenvalue problem for $\omega(\mu)$ is obtained:

$$[\widetilde{\boldsymbol{K}} - \omega^2 \widetilde{\boldsymbol{M}}]\widetilde{\boldsymbol{q}} = 0.$$

Direct Method determines real and imaginary components of propagations constants from given angular frequencies. In this approach, motion is governed by:

$$\boldsymbol{D}(\omega)\boldsymbol{q} = \boldsymbol{f} \quad \text{and} \quad \boldsymbol{D}(\omega) = \boldsymbol{K} - \omega^2 \boldsymbol{M}\,,$$

in which **D** is the dynamic stiffness matrix. The following eigenvalue problem for $\mu(\omega)$ is obtained:

$$\boldsymbol{\Lambda^H D}(\omega)\boldsymbol{\Lambda}\widetilde{\boldsymbol{q}} = 0.$$

Results and discussion

Three configurations were analyzed by varying m_2 (Table 1), while reference values of $m_1 = 1$, $a = 1$, and $k = 1$ were kept constant. Dispersion curves were obtained using the WFEM, allowing identification of the band gap frequency ranges for each case.

Table 1: Parameters for m_2

	Case 1	*Case 2*	*Case 3*
m_2 (kg)	2	5	10

Results indicate that increasing the mass ratio (α) significantly widens the band gaps in the periodic spring–mass system due to enhanced Bragg scattering from greater mass contrast. Dispersion curves for α = 2, 5, and 10 (Figs. 3–5) clearly illustrate this shift and enlargement.

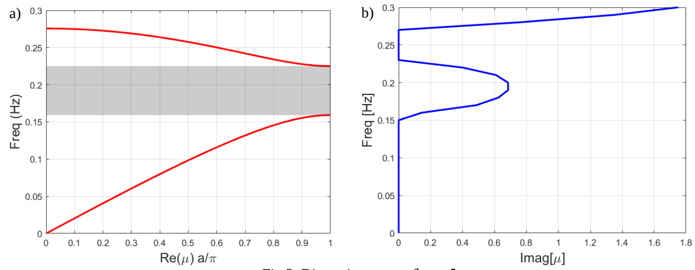

Fig.3: Dispersion curves for α=2

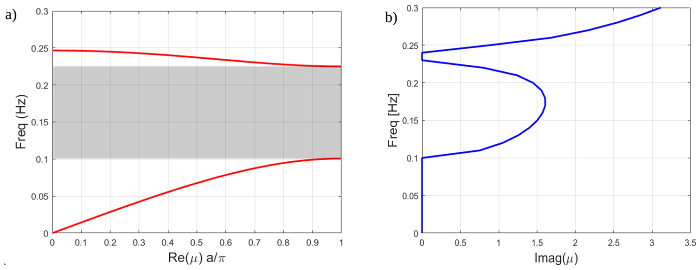

Fig.4: Dispersion curves for α=5

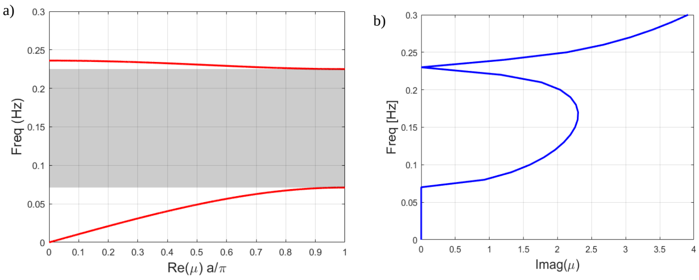

Fig.5: Dispersion curves for α=10

Table 2: Numerical resulsts for band gaps frequency range for 1D spring-mass system

Frequency (Hz)	**α = 2**	**α = 5**	**α =10**
ω_1	0.1592	0.1007	0.0712
ω_2	0.2251	0.2251	0.2251
Δω	0.0659	0.1244	0.1539

Table 3: Relative band gap increment with reference to α=2

α_5/α_2= 2.5	α_{10}/α_2= 5
88%	133%

Conclusion

Numerical results confirmed that the mass ratio (α) plays a crucial role in shaping and widening band gaps in 1D periodic spring–mass systems. Increasing α results in broader gaps and enhanced attenuation, establishing it as a key parameter for vibration control design. The findings also validate the WFEM as an efficient and reliable tool for analyzing and optimizing periodic structures aimed at passive wave suppression in engineering applications, particularly in railway tracks and other infrastructure subjected to dynamic loads.

Referencies

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*https://railroadrails.com/information/types-of-railway-tracks/