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II SIMPÓSIO DE MODELAGEM COMPUTACIONAL EM CIÊNCIA E TECNOLOGIA DO PIAUÍ

Wave Propagation in Periodic Railway Tracks: Lateral and Vertical Bending Analysis

Isadora Rodrigues de Souza (Universidade Federal do Piauí)
Simone dos Santos Hoefel (Orientadora, Universidade Federal do Piauí)
Laboratório de Métodos de Modelagem Computacional (LAMEC)
Email: irs08902@gmail.com, simone.santos@ufpi.edu.br

Introduction

Vibrations in railway tracks, resulting from wheel–rail interactions, can lead to structural damage, passenger discomfort, and environmental disturbances. Periodic structures offer a promising solution by generating bandgaps - frequency ranges that inhibit wave propagation - which can be classified into two types: Bragg scattering bandgaps (BSBG) and local resonance bandgaps (LRBG).

This work analyzes a periodic rail track with uniformly spaced supports, using the Wave and Finite Element Method, which combines finite element modeling with Floquet-Bloch theory to analyze periodic structures. The influence of variations in sleeper spacing and pad stiffness on the attenuation of vertical and lateral bending waves is investigated through dispersion curves, which describe the correlation between angular frequency and propagation constants (DOS REIS; DE SOUZA; HOEFEL, 2024).

Metodology

Figure 1 shows a rail track segment supported by sleepers and rail pads. The infinite periodic rail is modeled as an Euler-Bernoulli beam on linear spring supports, where a is the unit cell length (lattice constant),and k_v and k_l represent the vertical and lateral stiffness, respectively, as shown in Fig. 2. The mass and damping of the rail pad are ignored. Also, effects such as shear and rotational inertia are neglected.

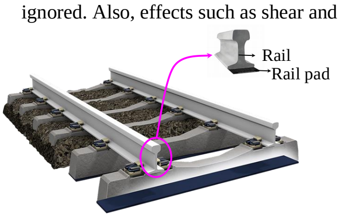

Fig. 1: Rail track with sleepers and rail pads. Fonte: Pandrol*

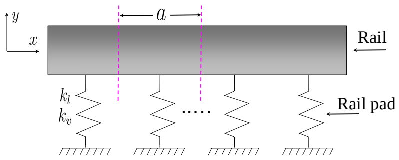

Fig. 2: Equivalent model of a periodic track structure.

The dynamic response of the substructure, assuming harmonic motion and negligible damping, is governed by the matrix equation:

$$\mathbf{K} - \omega^2\mathbf{M}\mathbf{q} = \mathbf{f},$$

where **K** and **M** are the stiffness and mass matrices, **q** is the vector of degrees of freedom of the considered structure, and **f** is the vector of nodal loads.

Applying the periodic Floquet-Bloch conditions to the degrees of freedom and forces at the boundaries of the unit cell:

$$\mathbf{q}_R = \mathbf{q}_L\lambda \quad \text{and} \quad \mathbf{f}_R = -\mathbf{f}_L\lambda,$$

with $\mathbf{q}_L$ and $\mathbf{q}_R$ are left/right boundary degrees of freedom, $\mathbf{f}_L$ and $\mathbf{f}_R$ the corresponding forces and $\lambda = e^{i\mu}$ the Bloch (Floquet) propagator ($\mu = ka$: propagation constant, k: wavenumber).

The relationship between the degrees of freedom vectors is given by $\mathbf{q}=\boldsymbol{\Lambda}\tilde{\mathbf{q}}$, where $\mathbf{q}$, $\tilde{\mathbf{q}}$ and the periodic transformation matrix $\boldsymbol{\Lambda}$ are defined as:

$$\mathbf{q} = [\, \mathbf{q}_L, \mathbf{q}_I, \mathbf{q}_R]^T, \quad \tilde{\mathbf{q}} = [\, \mathbf{q}_L, \mathbf{q}_I]^T, \quad \boldsymbol{\Lambda} = \begin{bmatrix} \mathbf{I} & 0 & \lambda\mathbf{I} \\ 0 & \mathbf{I} & 0 \end{bmatrix}.$$

In the absence of external forces, the force equilibrium condition is expressed in matrix form as:

$$\boldsymbol{\Lambda}^{\mathbf{H}}\,\mathbf{f} = 0,$$

where $\boldsymbol{\Lambda}^{\mathbf{H}}$ denotes the Hermitian transpose of the periodic transformation matrix $\boldsymbol{\Lambda}$.

By substituting the Floquet-Bloch transformation into the dynamic equation: $D(\omega)\mathbf{q} = 0$, with $D(\omega) = \mathbf{K} - \omega^2\mathbf{M}$ yields:

$$\boldsymbol{\Lambda}^{\mathbf{H}}\, D(\omega)\boldsymbol{\Lambda}\tilde{\mathbf{q}} = 0.$$

After some mathematical manipulations, this expression results in a quadratic eigenvalue problem of the following form:

$$\left(\underbrace{\begin{bmatrix} \mathbf{D}_{RL} & \mathbf{D}_{RI} \\ 0 & 0 \end{bmatrix}}_{D_0(\omega)} + \lambda \underbrace{\begin{bmatrix} \mathbf{D}_{LL} + \mathbf{D}_{RR} & \mathbf{D}_{LI} \\ \mathbf{D}_{IL} & \mathbf{D}_{II} \end{bmatrix}}_{D_1(\omega)} + \lambda^2 \underbrace{\begin{bmatrix} \mathbf{D}_{LR} & 0 \\ \mathbf{D}_{RI} & 0 \end{bmatrix}}_{D_2(\omega)}\right)\tilde{\mathbf{q}} = 0.$$

This approach enables the identification of propagating and evanescent modes, characterized respectively by the real and imaginary parts of the propagation constant.

*Disponível em: https://images.app.goo.gl/iAAVYQ95yi6rRKqk8

Results and discussion

The transmission behavior of vertical and lateral bending waves is investigated in the track structure. The influence of unit cell length and pad stiffness on bandgap characteristics is analyzed. The rail parameters are defined as: ρ = 7850 kg/m^3, EI_{zz} = 6.4e6 m^2, EI_{yy} = 1.07e6 m^2, while the rail pad parameters are: k_v = 3.5e8 N/m e k_l = 0.5e8 N/m.

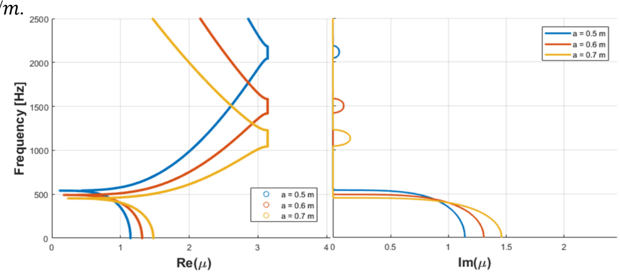

Fig. 3: Complex band structure for vertical bending waves.

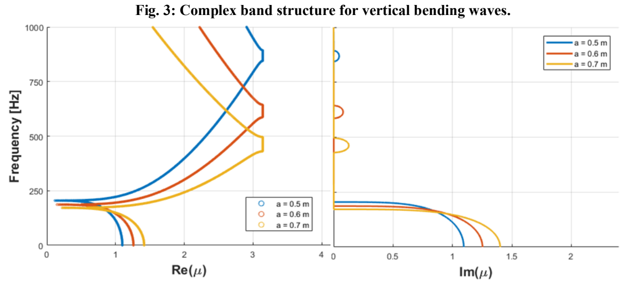

Fig. 4: Complex band structure for lateral bending waves.

Figures 3 and 4 respectively show the dispersion curves for vertical and lateral bending waves. For a lattice constant of 0.6 m, the results are in good agreement with those reported by Iqbal and Kumar (2024).

Table 1: Frequency range for vertical bending waves

a (m)	LRBG (Hz)	BSBG (Hz)
0.5	550.89	140.55
0.6	491.82	164
0.7	454.82	185.17

Table 2: Frequency range for lateral bending waves

a (m)	LRBG (Hz)	BSBG (Hz)
0.5	207.2	49.15
0.6	187,14	57.7
0.7	171.9	65.6

The complex band structures of vertical bending waves for different stiffness values is shown in Fig. 5.

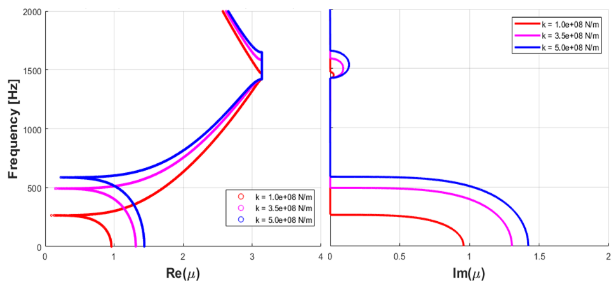

Fig. 5: Complex band structure of vertical bending waves for different pad stiffness values.

Conclusion

The results confirm that periodic rail tracks are effective in attenuating vertical and lateral bending waves through the formation of BSBG and LRBG. Parametric analyses showed that increasing the unit cell length shifts the BSBG to lower frequencies and reduces the width of the LRBG. In contrast, increasing the rail pad stiffness widens both band gaps and enhances attenuation performance.

References

IQBAL, Mohd; KUMAR, Anil. Analysis of bending waves and parametric influence on band gaps in periodic track structure. **Materials Today: Proceedings**, v. 113, p. 235-239, 2024.

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